Broadcasting Networks Planning

C. Mannino F. Rossi S. Smriglio

Dipartimento di Informatica e Sistemistica
Università di Roma “La Sapienza”

Dipartimento di Informatica
Università di L’Aquila

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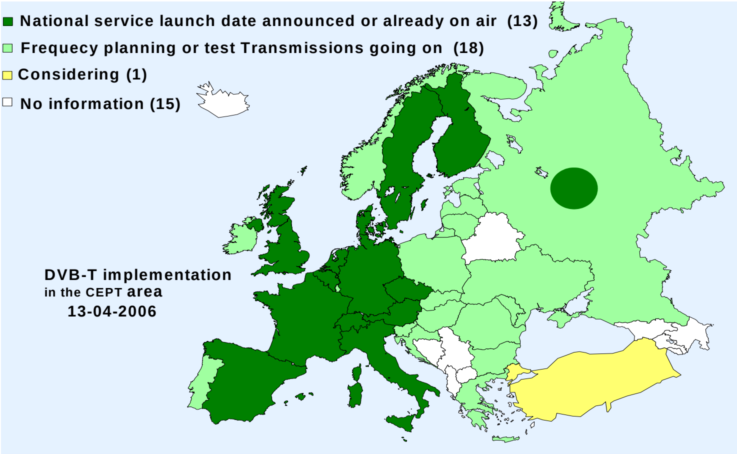
Terrestrial Digital Video Broadcasting: DVB-T

- DVB-T is considered the biggest technological novelty in television history
- Enhanced spectrum management
  - higher picture definition, higher audio quality
  - convergence with other media (radio, mobile telephony and Internet)
  - a larger number of programs (multiplex) per channel (from 1 to 5 digital programs vs. just 1 analog program)
  - pay-per-view, interactive TV

Market

- The European audiovisual sector was worth over 96 billion Euro at the end of 2000 (1.13 % of the GDP of EU)
- British public broadcaster will invest 6 billion Euro over the next 10 years on contents and services
- UK government start-up investment to cover 70% of the population was 256 million Euros (80 sites, year 2000)
- RAI investments for the digital deployment in 2006 amounts to 120 Million Euros
- rent out one channel within an Italian National multiplex yields 7-10 Million Euro/year (RAI had 40 requests in late 2004, one from China Central Television !)

Implementation status



Legislation	[1996, 2004]	Soft launch	[2000, 2005]
Full launch	[2000, 2006]	Analog switch off	[2008, 2012]

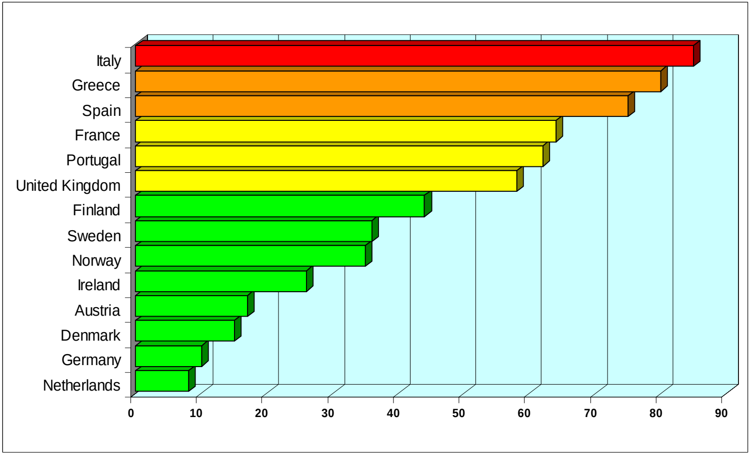
The Italian system

The most complex system in Europe:

- Largest number of operating transmitters
  over 23.000 vs 12.000 in France, 10.000 in Germany.
- Largest number of networks
  10 National networks and more than 700 local networks
- Frequency spectrum close to saturation
- About 70% of the spectrum owned by only three operators
  RAI 30.7 %, Mediaset 28.6 % and Telecom Italia 7.8 %
- Regulation Authorities do not have a precise map of the spectrum utilization

The Italian system

Share of terrestrial-only households:



The roadmap for the digital television

- The National analog networks cover almost all (95%) the population but use most of the spectrum, leaving no room for DVB-T
- European Regulation Authorities define new agreements for spectrum utilization [Geneva 2006]
- The digital deployment requires drawing frequencies from analog networks and switch them to DVB-T.

Players

- Regulation Authorities: guarantee market fairness
  - milestones, such as the date for definitive analog switch-off
  - requirements for the public broadcaster
  - incentives for customers.
- Broadcasters: minimizing costs and audience loss
- Customers: reduce inconvenience for set-up of digital equipments

The challenge for OR

Provide simulation and optimization models to support the deployment of the digital TV

- Public and commercial broadcasters:
  - Maximize coverage with current resources (transmitters, frequencies)
  - Cost/benefits analysis in purchasing new (expensive) frequencies.
  - Provide the management with alternative digital start-up configurations and possible network deployment
- Regulation Authorities:
  - Definition of optimized reference networks to establish regulations
  - Actions for market equilibrium (National Plan for Frequency Assignment)

Broadcasting network

Transmitter

Receiver

System Elements

Definition (Broadcasting Network)

A pair (T, Z), where:
T is a set of transmitters distributing a service over a target area
Z is a finite set of receivers located in the target area

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Receivers vs. Test-points

Antennae with decoder/TV sets: infinite number of possible locations

Standard technique: decomposition of the target area in a grid of testpoints (TPs)

Each testpoint represent the behavior of a receiver located anywhere inside it. Italy: 55000 to 197000 TPs.

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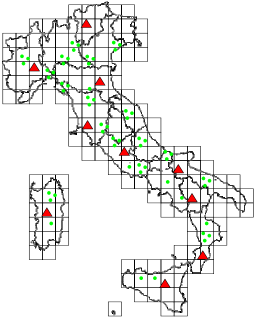

System Elements

Configuration of transmitter $t \in T$

Geographical location and antenna tower height: fixed

Transmission Frequency (channel):
$f_t \in \mathcal{F} = \{1, 2, \ldots, |\mathcal{F}|\}$

Radiation Pattern (W/m^2):
$P_t^d \in [P_t^{\min}, P_t^{\max}]$ power emitted by t in direction d

Antenna diagram:
radiation plot in the plain perpendicular to the antenna axis (horizontal antenna diagram)

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Antenna diagram

Discretization into 36 sectors

P_t^d is defined for $d = \{1, \ldots, 36\}$

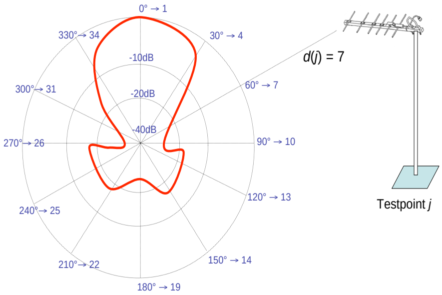
Power bounds

$P_t^d \in [P_t^{\min}, P_t^{\max}]$

Adjacency constraints

Power ratio between adjacent sectors must not exceed a threshold $\Delta \geq 1$

$\frac{1}{\Delta} P_t^{d-1} \leq P_t^d \leq \Delta P_t^{d+1}$



System Elements

Antenna diagram

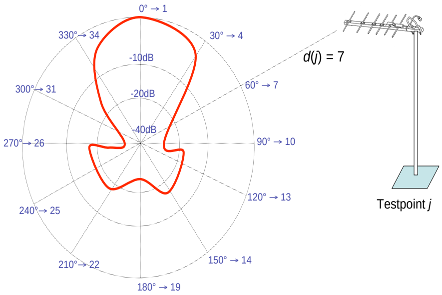
Front/rear constraints

Power ratio between opposite sectors must not exceed a threshold Γ

$$P_t^d \leq \Gamma P_t^{d+18}$$

Symmetry constraints

Power emitted in direction d must be equal to power emitted in direction $d + h$, for some d, h



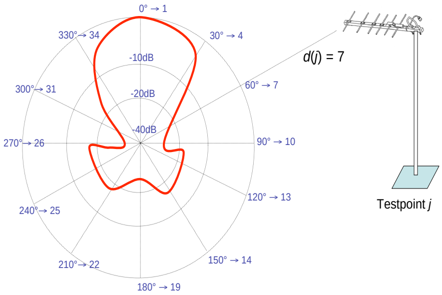
System Elements

Antenna diagram

For each $t \in T$, the set of technological constraints define a polyhedron $\mathcal{D}_t \subset \mathbb{R}^{36}$

A power vector $\boldsymbol{P}_t = (P_t^1, \ldots, P_t^{36}) \in \mathcal{D}_t$ and satisfying the power bounds is a feasible antenna diagram.

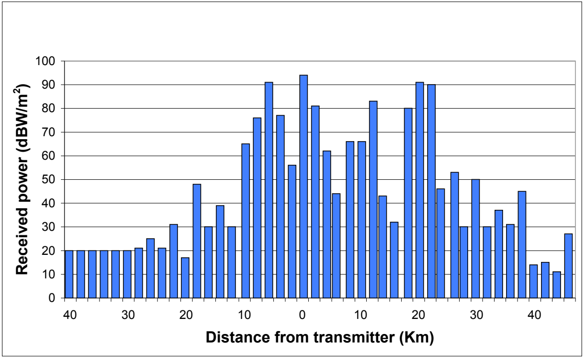
A transmitter is switched off if $\boldsymbol{P}_t = (0, \ldots, 0)$.



Received power

The power radiated from transmitter i at frequency f and received in a TP j depends on:

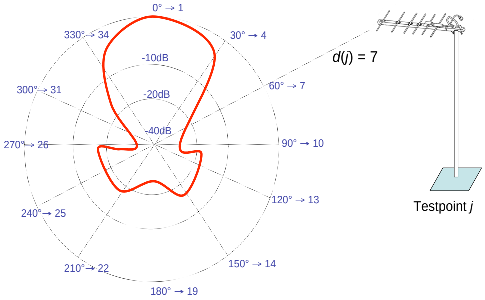
distance between i and j and terrain orography $\rightarrow$ fixed fading matrix



Received power

The power radiated from transmitter i at frequency f and received in a TP j depends on:

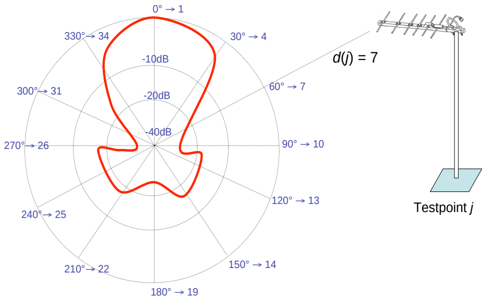
direction angle between i and j $\rightarrow$ fixed



Received power

The power radiated from transmitter i at frequency f and received in a TP j depends on:

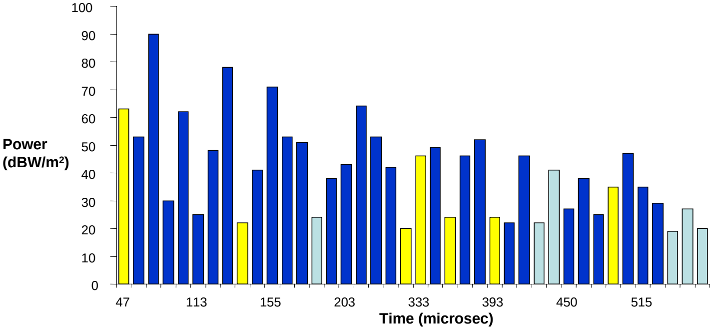
- orientation of the receiving antenna (chosen by the receiver)



$$P_{ij} = \underbrace{r_{ij} \cdot a_{ij}}_{\text{constant}} \cdot P_i^{d(j)}$$

Interference

The set of signals received in TP j is denoted by $T(j)$.
Among all received signals exactly one is selected as reference signal s by the receiver.



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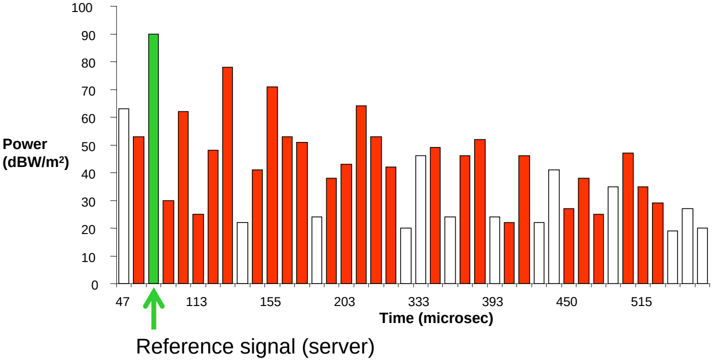
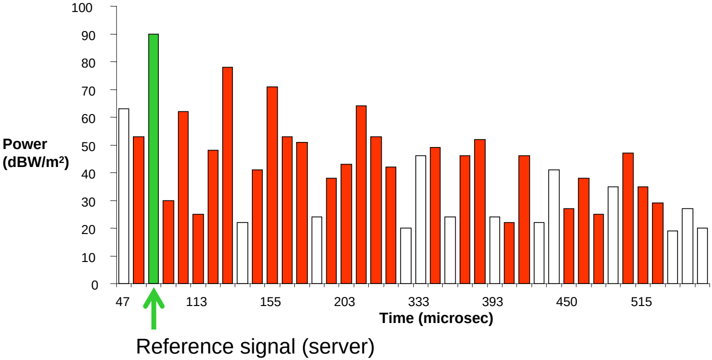


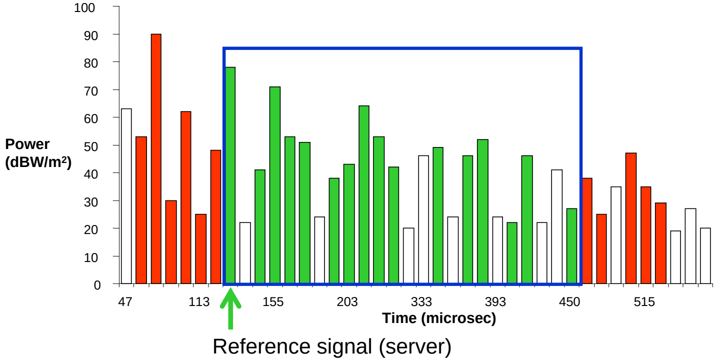
Interference

This choice determines the sets:
$W(j,s) \in T(j)$ useful contributions $\rightarrow \mathcal{P}(W(j,s))$
$I(j,s) \in T(j)$ interfering contributions $\rightarrow \mathcal{P}(I(j,s))$ (co-channel interference)
In analog broadcasting $W(j,s) = \{s\}$



Interference

In digital broadcasting signals different from s can be included in $W(j,s)$. The receiver consider all (isofrequency) signals within a Detection Window.
There exist only $|T(j)|$ significant positions of the DW.



Network coverage

Coverage of a testpoint

A TP j is covered $\Leftrightarrow$ there exists a reference signal s in $T(j)$ such that

$$\frac{\mathcal{P}(W(j,s))}{\mathcal{P}(I(j,s)) + \mathcal{N}} \geq \text{SIR (Signal-to-Interference Ratio)}$$

SIR inequality

$$\mathcal{P}(W(j,s)) = \sum_{i \in W(j,s)} P_{ij} \text{ and } \mathcal{P}(I(j,s)) = \sum_{i \in I(j,s)} P_{ij}$$

TP j is covered if

$$\sum_{i \in W(j,s)} a_{ij}(s) P_i^{d(j)} - \sum_{i \in I(j,s)} b_{ij}(s) P_i^{d(j)} \geq \sigma_j$$

The set of covered TPs is called coverage area C of the network

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Network configuration

coverage C depends on:

- Emission powers $P_i^d = 0,\ d \in \{0, 36\} \vee$ $\boldsymbol{P}_i \in (\mathcal{D}_i \cap [P_i^{\min}, P_i^{\max}])$
- Transmission frequencies $\boldsymbol{f} \in \{1, 2, \ldots, |\mathcal{F}|\}$
- Reference signals $\boldsymbol{s} \in \{0, 1\}^{|Z| \times |T|}$, $s_{tj} = 1$ if t is the server of j

The vector triple $(\boldsymbol{P}, \boldsymbol{f}, \boldsymbol{s})$ is called network configuration

$C(\boldsymbol{P}, \boldsymbol{f}, \boldsymbol{s})$ is the associated coverage area

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Network Planning

Power and Frequency Assignment Problem (PFAP)

Given

- a set T of transmitters
- a target area Z
- coefficients $a_{ij}(s)$, $b_{ij}(s)$, σ_j for all $i \in T$, $j \in Z$ and $s \in T(j)$
- the polyhedron $\mathcal{D}_i$ and power bounds $[P_i^{\min}, P_i^{\max}]$ for all $i \in T$
- a frequency domain $\mathcal{F}_i$ for all $i \in T$
- a revenue function $u(C)$ of the coverage

Find a network configuration $(\boldsymbol{P}, \boldsymbol{f}, \boldsymbol{s})$ such that $u(C(\boldsymbol{P}, \boldsymbol{f}, \boldsymbol{s}))$ is maximized.

Observation

PFAP is NP-hard in the strong sense (Mannino et al., 06)

Modeling PFAP: the SIR model

Decision variables

$$w_{jh}^{f} = \begin{cases} 1 \text{ iff TP } j \text{ is covered at frequency } f \text{under ref. signal } h \\ 0 \text{ otherwise} \end{cases}$$

$$z_{j}^{f} = \begin{cases} 1 \text{ iff TP } j \text{ is covered at frequency } f \\ 0 \text{ otherwise} \end{cases}$$

$$r_{i}^{f} = \begin{cases} 1 \text{ iff transmitter } i \text{ is assigned with frequency } f \\ 0 \text{ otherwise} \end{cases}$$

$$p_i^{d,f} \in \{0, [\tfrac{P_i^{\min}}{P_i^{\max}}, 1]\},$$ power fading of transmitter i at frequency f along direction d

Modeling PFAP: the SIR model

$$\max \sum_{j\in Z} \sum_{f\in\mathcal{F}} u_j z_j^f$$

$$\sum_{i\in W(j,h)} a_{ij}(h) p_i^{d,f} - \sum_{i\in I(j,h)} b_{ij}(h) p_i^{d,f} - M w_{jh}^f \geq \sigma_j - M \qquad j\in Z, h\in T(j), f\in\mathcal{F}_h$$

$$z_j^f - \sum_{h\in T(j)} w_{hj}^f \leq 0 \qquad j\in Z, f\in\mathcal{F}$$

$$(p_i^{1,f},\ldots,p_i^{36,f}) \in \mathcal{D}_i \qquad i\in T, f\in\mathcal{F}$$

$$(P_i^{\min}/P_i^{\max})\cdot r_i^f \leq p_i^{d,f} \leq r_i^f \qquad i\in T, f\in\mathcal{F}_i$$

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$\boldsymbol{w}, \boldsymbol{z}, \boldsymbol{r}$ binary

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Counting TPs just once:

$$\sum_{f\in\mathcal{F}} z_j^f \leq 1 \qquad j \in Z$$

$$\sum_{f\in\mathcal{F}} r_i^f \leq 1 \qquad i \in T$$

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$\max \sum_{j \in Z} \sum_{f \in \mathcal{F}} u_j z_j^f$

SIR Inequalities:

$$\sum_{i \in W(j,h)} a_{ij}(h) p_i^{d,f} - \sum_{i \in I(j,h)} b_{ij}(h) p_i^{d,f} - M w_{jh}^f \geq \sigma_j - M \qquad j \in Z, h \in T(j), f \in \mathcal{F}_h$$

TP coverage:

$$z_j^f - \sum_{h \in T(j)} w_{hj}^f \leq 0 \qquad j \in Z, f \in \mathcal{F}$$

Antenna Diagram:

$$(p_i^{1,f}, \ldots, p_i^{36,f}) \in \mathcal{D}_i \qquad i \in T, f \in \mathcal{F}$$

Variable Power Bounds:

$$(P_i^{\min} / P_i^{\max}) \cdot r_i^f \leq p_i^{d,f} \leq r_i^f \qquad i \in T, f \in \mathcal{F}_i$$

Counting TPs just once:

$$\sum_{f \in \mathcal{F}} z_j^f \leq 1 \qquad j \in Z$$

Transmitter frequency assignment

$$\sum_{f \in \mathcal{F}} r_i^f \leq 1 \qquad i \in T$$

$\boldsymbol{w}, \boldsymbol{z}, \boldsymbol{r}$ binary

On algorithms

Common practice: decomposition

1. Solve an instance of Power Assignment Problem
2. Solve an instance of Frequency Assignment Problem (often by a graph-based model)

Poor results!

But the SIR model is a difficult Mixed Integer Program:

- Large number of variables and constraints
- Coefficients in the SIR inequalities take values in $[1, 10^8]$
- The "big M" produces large integrality gap
- Equal frequency domains $\mathcal{F}_i$ of transmitters induce symmetry, i.e., many equivalent solutions can be obtained interchanging frequencies

Structure of frequency domains

1. All transmitters have the same frequency domain and $|\mathcal{F}| \geq 2$
2. Each transmitter has its own frequency domain
3. All frequency domains contain just one frequency

Case 1 and 2 yield the general PFAP.

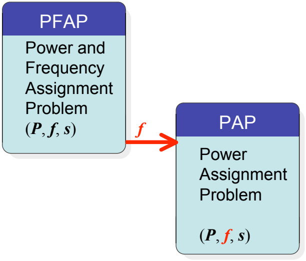
Case 1 arises in the definition of new general regulations by Public Authorities (White Book on Digital Television) (55,000 TPs, 480 transmitters and $|\mathcal{F}| = \{2, 3, 4\}$)

Case 2 is common among major broadcasters: some frequencies may be forbidden for general restrictions or licensing

Case 3 Interesting for all actors

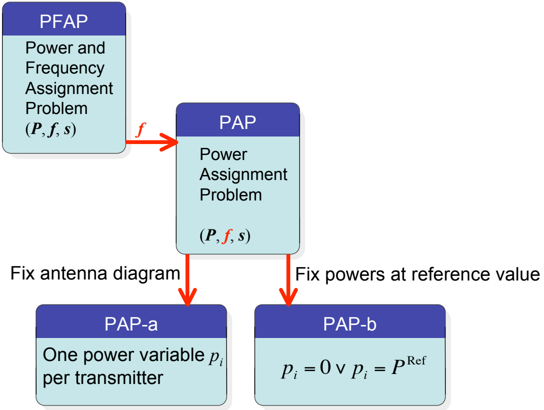
Problems Hierarchy

Case 3: since $|\mathcal{F}_i| = 1$, PFAP decomposes into $|\mathcal{F}|$ single frequency problems called Power Assignment problem.



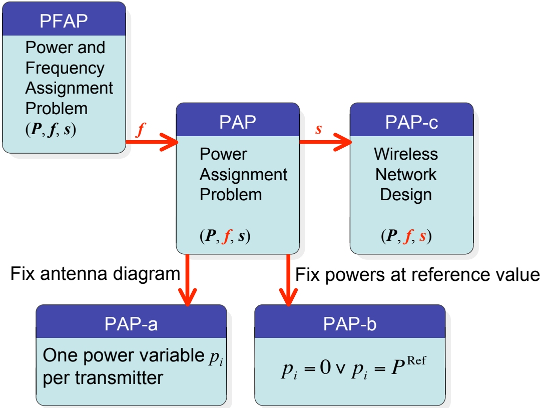
Problems Hierarchy

Three different versions of PAP occur in practice:
a) fixed antenna diagrams *b*) fixed diagrams and powers



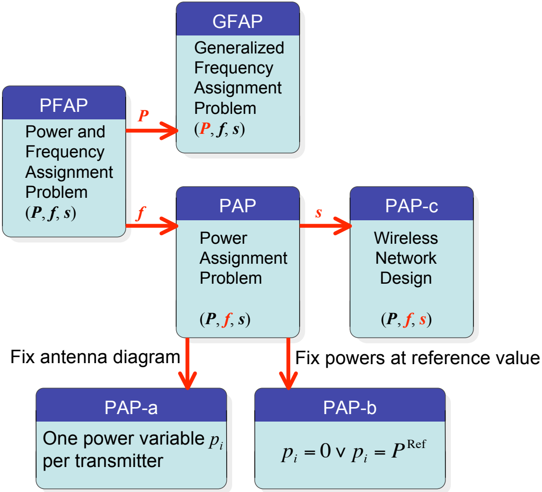
Problems Hierarchy

Three different versions of PAP occur in practice:
c) the reference transmitter is fixed in each testpoint



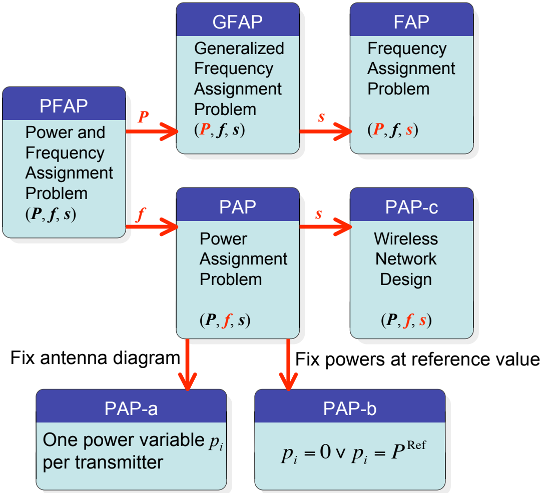
Problems Hierarchy

The planner might be interested in optimizing frequencies without changing emission powers:



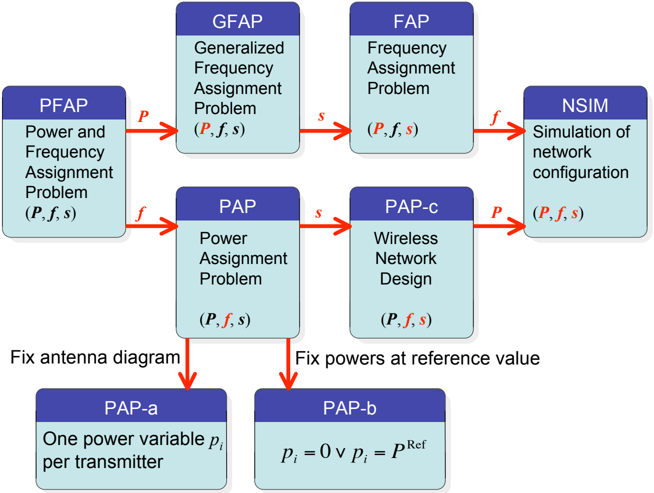
Problems Hierarchy

If also the reference transmitter is fixed for each TP, GFAP boils down to the classical Frequency Assignment Problem



Problems Hierarchy

At the bottom of the hierarchy there is the coverage evaluation procedure referred to as Network Simulation



Practical achievements

- AGCOM ’00: White Book on digital television (with A. Sassano)
- FUB ’06-’07 (A. Sassano):
    - international coordination;
    - analogue switch-off.
- RAI ’03: tools for A to D migration